ORIGINAL ARTICLE

LINEAR MIXED EFFECT MODELS AND APPLICATION IN ECONOMY

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Abstract

Background: North Africa has witnessed a political and economic evolution and very important strategic location. **Objective**: The aim of this work is the modeling and the simulation of gross domestic product (GDP), PPP (current international dollar) growth in North African countries from 1990 to 2017.**Methods:** Using mixed effects model theory. In our case, we will use linear mixed effects models for longitudinal data covering the last twenty seven years. **Results:** This model provides a good fit to describe the law of evolution of GDP (PPP based). **Conclusion**: The Linear Mixed Effects Models can help to characterize and to understand many complex linear economical processes.
***Keywords**: Linear mixed effects model, GDP (PPP based), North Africa, Longitudinal data.*

1. INTRODUCTION

North Africa has a very important strategic location for its proximity to Europe and its openness to sub-Saharan Africa. This part of the world has witnessed a political and economic evolution. The purpose of this work is the modeling of the Gross Domestic Product in current international dollar. An international dollar has the same purchasing power over GDP as United States which is the main economic standard for measuring economic output produced within a country. This modeling covers all North African Countries except Libya due to lack of data. The mathematical models allow the analysis and interpretation of the observed data because they describe the evolution law as a function of only a few parameters that can be statistically compared. For repeated measurements data, mixed-effects models offer a flexible and powerful tool in which population characteristics are modeled as mixed effects and unit-specific variation is modeled as random effects. Linear mixed-effects (LME) models [1, 2, 3, 4] and nonlinear mixed-effects (NLME) models [5, 6, 7] are widely used in longitudinal data analysis. The overall objectives of this paper is to make a linear mixed-effects model for describing GDP growth as a function of time (years), taking individualization (Inter-individual variation, Intra-individual variation) into account. Thus, the linear mixed-effects model introduced in this paper provides a good fit for data.
Finally, the analysis of this research was accomplished with the lme4 package [8] for R statistical software.

2. Generalized linear mixed-effect models

Suppose that a study is based on N individuals and that we seek to build a global model for all thecollected observations for the N individuals. Inspired from the work [9], we denote y_{ij} the j observation taken of individual i and $t_{ij}^{(1)}, \dots, t_{ij}^{(m)}$ the values of the m explanatory variables for individual m .

If we assume that the parameters of the model can vary from an individual to another, then for any subject i , $1 \le i \le N$, the model is:

$$y_{ij} = b_{i0} + b_{i1} t_{ij}^{(1)} + b_{i2} t_{ij}^{(2)} + \cdots + b_{im} t_{ij}^{(m)} + \epsilon_{ij}, \qquad 1 \le j \le n_i \qquad (2.1)$$

To begin with, suppose that each individual parameter b_{ik}can be broken down into a fixed component $\beta_k$and an individual component r_{ik} additively:

$$b_{ik} = r_{ik} + \beta_k \qquad (2.2)$$

Where b_{ik} represents the deviation of r_{ik} from the value β_k in the population for individual i and r_{ik} is a random variable normally distributed with mean 0. Using this parameterization, the model becomes:

$$y_{ij} = \beta_{i0} + \beta_{i1}t_{ij}^{(1)} + \beta_{i2}t_{ij}^{(2)} + \cdots + \beta_{im}t_{ij}^{(m)} + r_{i0} + r_{i1}t_{ij}^{(1)} + \cdots + r_{im}t_{ij}^{(m)} + \epsilon_{ij}, 1 \le j \le n_i \qquad (2.3)$$

We can then rewrite the model in matrix form:

$$y_i = T_i\beta + T_i r_i + \epsilon_i, \qquad (2.4)$$

Where:

$$y_i = \begin{pmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{in_i} \end{pmatrix}, \ T_i \begin{pmatrix} 1 & t_{i1}^{(1)} & \cdots & t_{i1}^{(m)} \\ 1 & t_{i2}^{(1)} & \cdots & t_{i2}^{(m)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & t_{in_i}^{(1)} & \cdots & t_{in_i}^{(m)} \end{pmatrix}, \ \beta = \begin{pmatrix} \beta_{i1} \\ \beta_{i2} \\ \vdots \\ \beta_{im} \end{pmatrix}, \ r_i = \begin{pmatrix} r_{i1} \\ r_{i2} \\ \vdots \\ r_{im} \end{pmatrix}, \text{ and } \epsilon_i = \begin{pmatrix} \epsilon_{i1} \\ \epsilon_{i2} \\ \vdots \\ \epsilon_{in_i} \end{pmatrix},$$

where y_i is the n_i vector of observations for individual i, T_i is the $n_i \times d$ design matrix (with$d = m + 1$), β is a d-vector of fixed effects (i.e. common to all individuals of the population), r_i isa d-vector of random effects (i.e. specific to each individual) and ϵ_i is a n_i-vector of residual errors.

The model is called linear mixed effects model because it is a linear combination of fixed and randomeffects. The random effects are assumed to be normally distributed in a linear mixed effects model:

$$r_i \sim \mathcal{N}(0_d, \Omega), \qquad (2.5)$$

Where Ω is the $d \times d$ variance-covariance matrix of the random effects. This matrix is diagonal if the components of r_i are independents.
The vector of residual errors $\epsilon_i$is also normally distributed:

$$\epsilon_i \sim \mathcal{N}(0, \Sigma_i), \qquad (2.6)$$

3. MODELING

3.1 Data

The data used in this paper comes from the World Bank and the International Monetary Fund. They describe the evolution of the Gross Domestic Product (current international dollar) in the countries of North Africa during the last twenty-seven years.

Our data has 60 rows and 3 columns, while the columns containing the grouping factor (indicating the subject), the predictor(s) and the response. In our case, the grouping factor is Country, while that the second contains the Year (the predictor) and the third its GDP in current international Dollar (the response). The plot of the data show in Figure 1.

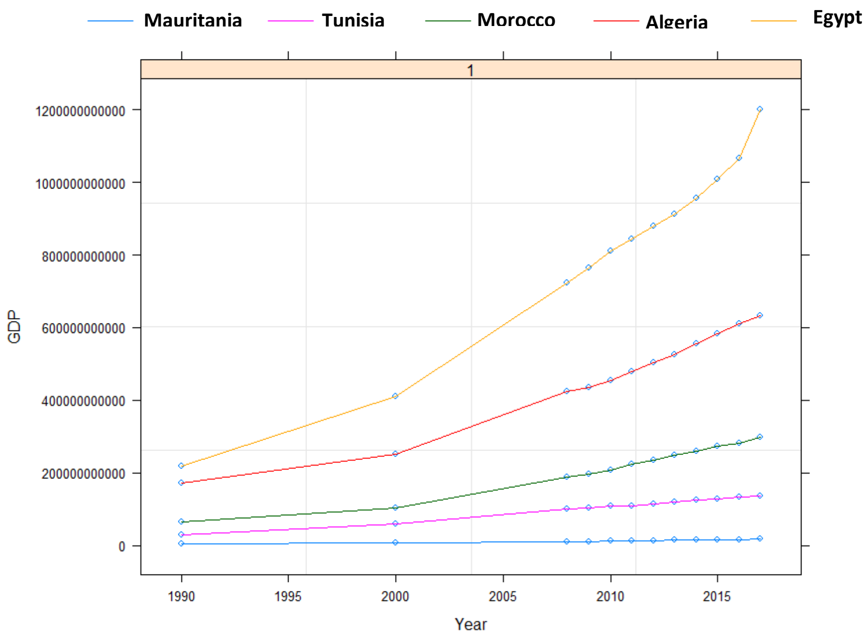

Figure 1: The figure presents Gross Domestic Product (GDP, current $) versus Year.

3.2 Models

The mixed effects model combines a model for the fixed effects and a model for the random effects.
Let us see some possible combinations.

3.2.1 Model 1

For the first proposed linear mixed effects model, we assume that gross domestic product (GDP) and growth rate (i.e. intercept and slope) may depend on the individual, we have 5 individuals (5 countries $i = 1,2, \dots ,5$),

$$y_i = b_{i0} + b_{i1} t_{ij} + \epsilon_{ij}, \qquad 1 \leq j \leq 27 \qquad (3.1)$$

Where t_{ij} is a regression variable representing "Year".

Table 1: The table presents the fixed effects results of fitting model 1.

Parameter	Estimate	Std. Error	t value	Correlation
β_0	-26299290120371	3453948510583	-7.514	
β_1 13247968623	1721065234	7.613		

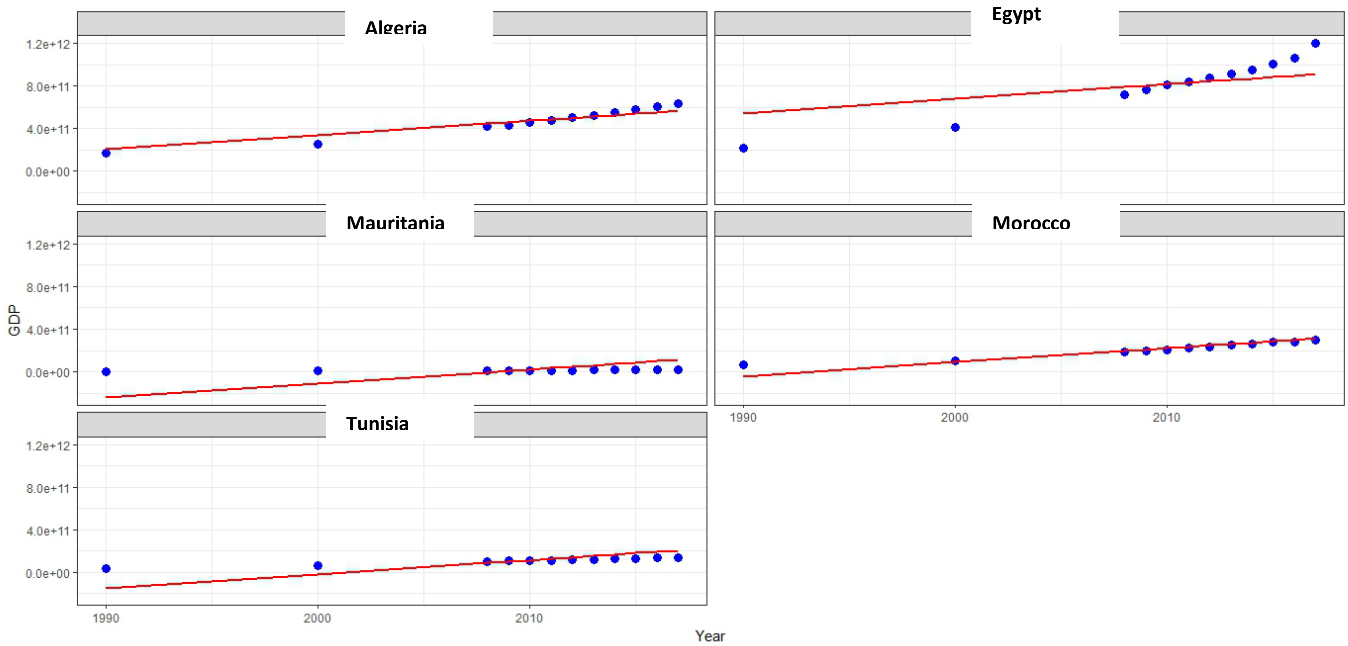

Figure 2: The figure presents the predicted Gross Domestic Product (**GDP)** versus observed **GDP**.

3.2.2 Model 2

We assume that the intercept randomly varies between individuals and the GDP increases with the same rate for all the individuals. The mathematical representation of this model is:

$$y_{ij} = b_0 + b_1 t_{ij} + r_{i0} + \epsilon_{ij}, \quad 1 \leq i \leq 5 \ \text{and} \quad 1 \leq j \leq 27 \qquad (3.2)$$

Where t_{ij} is a regression variable represents "Year".

Table 2: The table presents the fixed effects results of fitting model 2.

Parameter	Estimate	Std. Error	t value	Correlation
β_0	-26299290120359	3500173713954	-7.514	-0.999
β_1 13247968623	1740248929	7.613		

3.2.3 Model 3

In this model, we assume that the intercepts are the same for all individuals but different growth rates for individuals (we put a random effect on the slopes). The mathematical representation of this model is:

$$y_{ij} = b_0 + b_1 t_{ij} + r_{i1} t_{ij} + \epsilon_{ij}, \quad 1 \le i \le 5 \text{ and } \quad 1 \le j \le 27 \qquad (3.3)$$

Table 3: The table presents the fixed effects results of fitting model 3.

Parameter	Estimate	Std. Error	t value
β_0	12898374004	16898205283	0.763

3.3 Comparing linear mixed effects models

Bayesian information criterion **(BIC)** [10] and Akaike's information criterion **(AIC)** [11] are used to compare several models.

Table 4: The table presents the results of ANOVA.

Model	AIC	BIC	logLik	deviance
Model 1	3240.6	3253.2	-1614.3	3228.6
Model 2	3238.3	3246.6	-1615.1	3230.3
Model 3	3134.5	3145.0	-1562.3	3124.5

The best model, according to **(AIC)** and **(BIC)**, is the Model 3 that assumes same fixed intercept and a random slope. The plot of individual predicted GDP versus observed GDP (Figure 3):

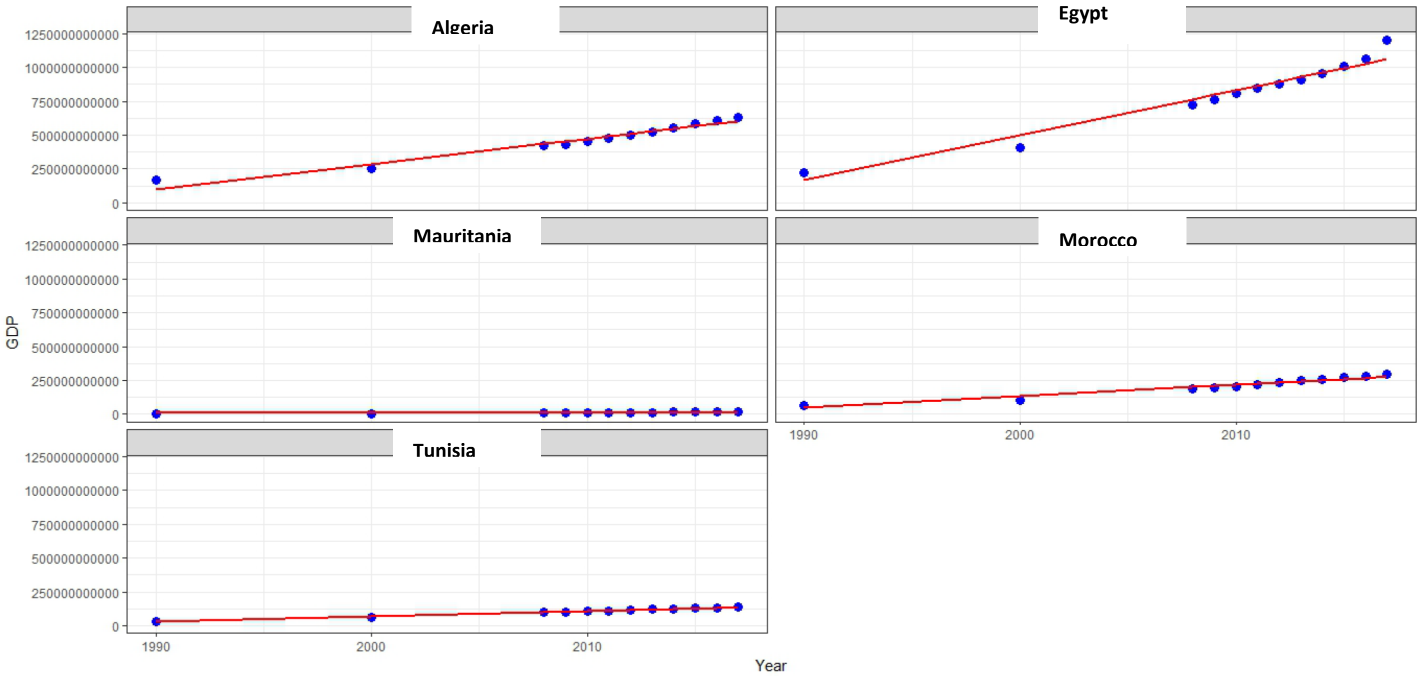

Figure 3: The figure presents the predicted Gross Domestic Product (**GDP)** versus observed **GDP**.

3. CONCLUSION

The modeling and simulation of the Gross Domestic Product (GDP) data using Linear Mixed-Effects Models proved to be successful. We can say that the Linear Mixed Effects Models can help to characterize and to understand many complex linear economical processes. This study has shown the effectiveness of the linear mixed effects models as a new approach to explain GDP data. The final proposed model (model 3) has proven good estimations for all parameters in the model.

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List of acronym:

GDP: Gross Domestic Product is a monetary measure of the market value of all the final goods and services produced in a period of time, often annually or quarterly. Nominal GDP estimates are commonly used to determine the economic performance of a whole country or region, and to make international comparisons.
AIC: The Akaike information criterion: is an estimator of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Thus, AIC provides a means for model selection.
BIC: The Bayesian information criterion is a criterion for model selection among a finite set of models; the model with the lowest BIC is preferred. It is based, in part, on the likelihood function and it is closely related to the Akaike information criterion (AIC).

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