ORIGINAL ARTICLE

A COOPERATIVE GAME THEORY APPLICATION IN CHICKS BROOD FOOD ALLOCATION BY USING SHAPELY VALUE METHOD IN BAD YEARS DATA

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ABSTRACT

Background: Game theory as well as cooperative game theory has played a vital role in many fields of research since its introduction within the early twentieth century. Coined by Shapley (1953), this one-point solution concept introduced in his paper has some desirable properties called efficiency, anonimity, dummy, and additivity. **Methods:** We study food allocation in chick broods from the attitude of cooperative theory of games. We would like to explore whether or not food distribution data fit into the known solution concepts of cooperative game theory to create an economic temperature within the biological field. A primary approach to be handled is that the incontrovertible fact that within the chick brood data we only see the solutions, while the starting position, the game, isn't immediately clear. In and of itself we'd like to reconstruct the game from the solutions given. A second approach is that by using the answer concepts Shapley value we would like to investigate which of those fits best. **Results:** The restricted structure of the brood datasets also enables the Shapley value solution concept to give a reasonable fit within a reasonable time. **Conclusion:** The better the game fits the solutions, the more we can trust the resulting system. Shapley value solution concept to tackle the chick brood food allocation problem. We also successfully translate the biological problem of chick's food allocation into a cooperative game approach using various techniques known in literature.

Keywords: *Player, Core, Marginals, Coalitions, Hatching, Survival rate, Parental, Standard approach, Restricted Approach.*

1. INTRODUCTION

Game theory is an autonomous discipline that is used in applied mathematics, social sciences, most remarkably in mathematics, economics, as well as in computer science, biology, engineering, international relations, philosophy, and political science [1]. It was displayed in economics and mathematics to measure economic behaviors including behaviors of firms, markets, and consumers. The cooperative game theory can be contemplated as a modeling procedure that is used to analyze and explain the actions of all players joined in competitive situations and to compare and determine the relative optimality of distinct strategies. The first inspect of games in terms of economics was by Cournot on pricing and production but Neumann (1944) is considered as the founder of the modem game theory [2]. Coined by Shapley (1953), this one-point solution concept introduced in his paper has some desirable properties called efficiency, anonymity, dummy, and additivity. A solution is efficient if it assigns to every game an allocation in such a way that the sum of every marginal contribution of each player will be equal to the value of the grand coalition [3, 4]. Recent works by Forbes (2005, 2007) is employing a financial tool to the study of parental investment in chick, as normally the foremost important investment any organism makes is in its offspring [5, 6]. In cooperative game theory, it can be said that the set of bounded computational capacity of equilibrium payoffs carries only one valuation, that the valuation of the game with penalty approaches the valuation of the one-shot game as the penalty goes to zero [7].

2. METHODS

2.1 Notations:

eggs: Shows how many eggs are firstly available in total before the hatching period.
c: In day 1, denotes the number of core eggs (eggs that are hatched in day 1) inside one brood.
m: The number of marginal eggs (eggs that are left/not yet hatched in day 1) in one brood.
brood: Shows how many broods that are available in total.
m_1**:** The number of marginal that are hatched on the first day after the core.
m_2**:** The number of marginal that are hatched on the second day after the core.
m_3**:** The number of marginal that are hatched on the third day after the core.
total: In day 8, shows how many chicks that are continue living after one week of feeding.
d8 val: The average number of all chick (both core and marginal) which survive until one week of feeding (i.e. day 8).
m av: The average of the marginal chick' survival rate which we will use to fill the coalition value for the marginal later in our method.
c val: The survival rate of each chick.
m_1 **val**: The survival rate of starting from the core chick.

m_2 **val**: The survival rate of the first marginal chick.
m_3 **val**: The survival rate of until the third marginal chick.
v(S): The value of the coalition.
A : A matrix with real entries a_{ij}.
x : A column vector with real entries x_i.
x^S : The payoff vectors.
$\{x_1, x_2\}$: The sequence of vectors x_1, x_2
x^* : The optimal value of vector **x**
$\emptyset$: Shapley value

2.2 The Chick Brood Data in Bad Years

The chick are puppies who have hatched together on the first day of nesting. Instead, chick hatch for more than a day and are identified as marginal chick. Parental choices for the number of eggs hatching on the first day may be based on their experiences during the early hatching period, or their instincts about the weather and food conditions near the nest. They hit their marginal according to their core every day. Therefore, if you have 2 core chick and 3 margins, the total hatch time is 4 days (1 day for all cores and 3 days for each margin). The raw data in Table 1 below will give you an idea of the number of chickens and chickens that one chick can have, and the number of chick available to chickens and chick. Be found. You can also see how many chicks died in a bad week (from day 1 to day 8), so you can see if there are any differences in how parents are assigned.

Table 1: Table presents the bad year data.

eggs	Day 1			Day 8					
	c	m	brood	c	m	m_1	m_2	m_3	total
6	1	0	6	6	0	0	0	0	6
40	1	1	20	22	17	16	1	0	35
100	1	2	33	39	48	26	22	0	63
116	1	3	29	46	56	27	22	6	68
50	2	0	24	46	0	0	0	0	46
153	2	1	51	104	35	34	0	0	121
292	2	2	73	136	61	44	17	0	168
55	2	3	11	19	13	9	3	1	29
54	3	0	18	44	0	1	0	0	44
331	3	1	83	201	25	25	0	0	198
40	3	2	8	23	1	1	0	0	22
28	4	0	7	21	0	0	0	0	21
15	4	1	3	9	1	1	0	0	10

As we've seen within the above data, one brood can contain at maximum 4 core chick, while on the opposite hand, it may also contain at maximum 3 marginal. While the whole number of chick is seven (i.e. there are seven players within the game), we don't have data with four core and three marginal at the identical time. The largest brood we've consisted of 5 chicks, either 2 core with 3 marginal, 3 core with 2 marginal, or 4 cores with 1 marginal. Note that from the story of chick the fogeys will feed the chick that beg louder, which usually are the core chick.

Therefore, in a method to calculate the Shapley value, we fancy assuming that the feeding process will always start with the core chick, while the marginal 'fight' over the remaining food after the cores are being fed. This assumption is described later during calculations and experiments of our method. In theory, we also cannot have only marginal without having the core, or having the third and/or the second marginal without having the primary one. But this can be happening in a number of the brood data since there's a break that the egg is missing or being destroyed during the hatching period of the core chick, to not mention the chick that's directly dead after born, leaving only the marginal within the brood.

The same case is additionally happening for the marginal. However, later we are going to see that our method excludes this sort of missing data from the calculations, and considers only the feasible coalitions. As a result of being born on different days where the marginal are hatched on a daily basis after the cores, we expect our brood data have a particular property: there exist different weights between core and marginal chick. this can be because the core and also the marginal chick may value their food in numerous ways. We predict that everyone the core chick's **c** will value their food within the same manner since they're hatched on an identical day (thus could also be as strong as each other) and also the parents consider them to be equally important to continue the family legacy.

As a result, the weights difference between the core chick is incredibly small or is ignored. In other words, we assume that competition between the core chick in one brood doesn't exist. However, there exist different weights between the core and also the marginal, in addition to between all the marginal, because the marginal m_i, $i = 1$ to 3, are born consecutively on i days after the core. Note that this weight isn't a body-mass index but a further number that represents how the chick may value their food. To be ready to build a coalition model for the brood food allocation data during the great and also the bad years, we firstly we define an XY-brood game where X and Y denote the quantity of core and marginal children respectively. As input for the chick brood food allocation model, we use the type of survival rate data A and also the importance weights data I which shows what percentage times a particular style of brood (i.e. core and marginal coalition) appears within the game.

Table 2: Table presents the average of the Survival Rate for XY-brood type during the Bad Years.

				A				I
c	**m**	**c val**	**m av**	m_1 **val**	m_2 **val**	m_3 **val**	**d8 val**	
1	0	1.000	0	0	0	0	1.000	6
1	1	1.000	0.850	0.850	0	0	1.850	20
1	2	0.848	0.774	0.839	0.710	0	2.397	33
1	3	1.000	0.644	0.931	0.759	0.453	2.897	29
2	0	0.958	0	0	0	0	1.917	24
2	1	0.931	0.686	0.686	0	0	2.549	51
2	2	0.836	0.418	0.603	0.233	0	2.507	73
2	3	0.818	0.433	0.900	0.300	0.200	2.936	11
3	0	0.815	0	0	0	0	2.444	18
3	1	0.763	0.309	0.309	0	0	2.598	83
3	2	0.917	0.063	0.125	0	0	2.875	8
4	0	0.750	0	0	0	0	3.000	7
4	1	0.750	0.333	0.333	0	0	3.333	3

Having the coalitions S, what value can we choose to be the value of the coalition v(S)? Since we have the average of the survival rate **A** for each off-springs in every XY-brood, taking into account its importance weight **I** (i.e. how many times the XY-brood data occur), we can take these values as the value of the coalitions.
But how to put the 'right' value into the 'right' coalition? Here we propose a coalition procedure which we adapt from the Shapley value procedure.

I. Suppose we have an XY-brood game during the bad years, with maximum 7 players (i=1 to 7), consists of maximum 4 core (i=1 to 4) and maximum 3 marginal players (i=5 to 7). Firstly, we define all possible coalitions of (X+Y) players where there are X core players and Y marginal.

   For example, if we take the bad years 21-brood data where there are two core players and one marginal player, it is possible to have coalitions of every single player, coalition between the core players, coalitions between each core with the marginal, and coalition of all the three players. In other words, for the bad years 21-brood, the possible coalitions of the 3 players are: $\{1\}, \{2\}, \{5\}, \{1,2\}, \{1,5\}, \{2,5\}$ and $\{1,2,5\}$.

II. We then re-translate every possible coalition into the number of core and marginal players in a brood.

   For example, coalition {1} and {2} is when we only have one core player in a brood, without having any marginals; i.e. the 10-brood. Thus, for the single core player coalitions, we will consider the 10-brood game. Now we do the same translations for the other coalitions: consider 20-brood game for the coalition between the core players {1,2} 11-brood game for the coalitions of each core player with the marginal $(i.e. \{1,5\}$ and $\{2,5\})$ and simply 21-brood game itself for the coalition of all three players {1,2,5}. As an exception, for the marginal player coalition {5} in 21-brood data, we cannot take the 01-brood into account, since by definition, no marginal can be hatched before having the core hatched. Thus we do not need to consider this kind of brood in the procedure.

III. Now we continue by looking at Table 2 for the bad years, in order to see the average of the survival rate in the corresponding XY-brood data, which we need to consider for each coalition.

   As an example, to fill in the coalition value v(S) of the single core player coalitions {1} and {2}, we take the average of the survival rate for this core chick in 10-brood, which is 1.000 (see **c val** on the table).

For the marginal player coalition {5} which is an exception, we choose to take the survival rate of the marginal on its first appearance in the brood datasets, which is the m_1 **val** value on table: 0.931. Note that this m_1 **val** is equal to the **m val** as we only have one marginal in 10-brood game.
We will provide two tables showing the chicks' average survival rates using the two approaches: the standard and the restricted approach. Note that these two table are adapted from Table 2.

Table 3: Table presents the survival rates of the players in the existing coalitions (Standard Approach, Bad Years).

XY	Possible Coalitions S	x^S			
		1 to 4	5	6	7
10	{1}, {2}, {3}, {4}	1.000	0	0	0
11	{1,5}, {2,5}, {3,5}, {4,5}	1.000	0.850	0	0
12	{1,5,6}, {2,5,6}, {3,5,6}, {4,5,6}	0.848	0.839	0.710	0
13	{1,5,6,7}, {2,5,6,7}, {3,5,,76}, {4,5,6,7}	1.000	0.931	0.759	0.200
20	$\{i,j\}, \forall i,j = 1\ to\ 4, i < j$	0.958	0	0	0
21	$\{i,j,5\}, \forall i,j = 1\ to\ 4, i < j$	0.931	0.686	0	0
22	$\{i,j,5,6\}, \forall i,j = 1\ to\ 4, i < j$	0.836	0.603	0.233	0
23	$\{i,j,5,6,7\}, \forall i,j = 1\ to\ 4, i < j$	0.818	0.900	0.300	0.100
30	$\{i,j,k\}, \forall i,j,k = 1\ to\ 4, i < j < k$	0.815	0	O	0
31	$\{i,j,k,5\}, \forall i,j,k = 1\ to\ 4, i < j < k$	0.763	0.309	0	0
32	$\{i,j,k,5,6\}, \forall i,j,k = 1\ to\ 4, i < j < k$	0.917	0.125	0	0
40	{1,2,3,4}	0.750	0	0	0
41	{1,2,3,4,5}	0.750	0.333	0	0

Table 4: Table presents the survival rates of the players in the existing coalitions (Restricted Approach, Bad Years).

XY	Possible Coalitions S	x^S						
		1	2	3	4	5	6	7
10	{1}	1.000	0	0	0	0	0	0
11	{1,5}	1.000	0	0	0	0.850	0	0
12	{1,5,6}	0.848	0	0	0	0.839	0.710	0
13	{1,5,6,7}	1.000	0	0	0	0.931	0.759	0.200
20	{1,2}	0.958	0.958	0	0	0	0	0
21	{1,2,5}	0.931	0.931	0	0	0.686	0	0
22	{1,2,5,6}	0.836	0.836	0	0	0.603	0.233	0
23	{1,2,5,6,7}	0.818	0.818	0	0	0.900	0.300	0.100
30	{1,2,3}	0.815	0.815	0.815	0	0	O	0
31	{1,2,3,5}	0.763	0.763	0.763	0	0.309	0	0
32	{1,2,3,5,6}	0.917	0.917	0.917	0	0.125	0	0
40	{1,2,3,4}	0.750	0.750	0.750	0.750	0	0	0
41	{1,2,3,4,5}	0.750	0.750	0.750	0.750	0.333	0	0

With the coalition procedure as described before, we got this coalition table for 21-brood game during the bad year's period:

Table 5: Table presents the 21-brood game, Bad years.

S	**{1}**	**{2}**	**{5}**	**{1,2}**	**{1,5}**	**{2,5}**	**{1,2,5}**
$v(S)$	1.000	1.000	0.850	1.917	1.850	1.850	2.549

We build a Shapley value calculation table for the bad years 21-brood data taking into account every possible orders of the players as follows.

Table 6: Table presents the possible orders for 21-brood game, Bad years.

Possible Orders	Player			Total
P	1	2	5	
1-2-5	1.000	0.917	0.632	
1-5-2	1.000	0.699	0.850	
2-1-5	0.917	1.000	0.632	
2-5-1	0.699	1.000	0.850	
5-1-2	1.000	0.699	0.850	
5-2-1	0.699	1.000	0.850	
∅	0.886	0.886	0.777	2.549
Observ.	0.931	0.931	0.686	2.549

Removing the unfeasible ones, we will get result:

Table 7: Table presents the feasible orders for 21-brood game, Bad years

Feasible Orders	Player			
P	1	2	5	Total
1-2-5	1.000	0.917	0.632	
2-1-5	0.917	1.000	0.632	
Ø	0.9585	0.9585	0.632	2.549
Observ.	0.931	0.931	0.686	2.549

In the case of taking all possible orders of 21-brood data, the observation results for the average of the core chicks' survival rate are indeed higher, compare to the results from the Shapley value calculations. The other way around happens for the marginals: if we consider all possible orders into account, the marginal's survival rate is 12% higher in calculation rather than in reality.

3. RESULTS AND DISCUSSION

We could say that marginal chicks in this case of bad years 21-brood data, seem to be 'stronger' in calculation rather than in the real world. In other words, the parental favoritism exists also in the bad year; and clearly is even worse in the 21-brood game. However, taking the feasible orders show a little bit different result. We might want to see whether this is always the case for every bad year's brood data. However, calculating the same procedures by hand on every XY-brood data during the bad years will be time consuming and not very effective; especially when we arrive on the larger broods like 23-brood game or 32-brood game.

By looking at these tables above we found that when we consider every possible orders into the Shapley value calculation for the bad years 21-brood data, the two CG-solution properties are also satisfied.

A key dimension of any investment decision including what proportion to speculate in offspring is a way to balance risk and reward that portfolio theory offers a broad set of analytical tools. An initial complexity for the biologist introduces in a way to translate the economic models into a biological patent. The tool he used is named financial beta and is well-known to the study of parental investment, derived from the capital asset pricing model of recent portfolio theory. Beta provides a measure of the volatility within the price of an asset for a wide market or index. Forbes suggested that the reproductive returns from individual brood structures (e.g., mean edging success in an exceedingly given year) may well be usefully equated to a private asset, which means population reproductive success may well be equated to the market as an entire. There is another study conducted by Alex Kacelnik, Peter A. Cotton, Liam Stirling, and Jonathan Wright which use the evolutionary theory of games to review Food Allocation among Nestling Starlings, drawing attention on Sibling Competition and also the Scope of Parental Choice. Chick feeding in chick is commonly viewed as a chief example of evolutionary conflict. this can be because the nestlings may benefit by inducing the parent to speculate more within the current brood compared to future ones. additionally, each nestling should benefit by obtaining a greater fraction of the full brood provision than would be optimal for the parent. Current theory suggests that at evolutionary equilibrium, the intensity of signalling (i.e. begging) by the chick should allow the fogeys to spot each chick's needs and to allocate more food to the one that gives the steepest marginal fitness gain per unit of parental resources.

4. CONCLUSION

This paper gives a mild and gentle intro to cooperative game theory in chick's brood food allocation since the modeling and calculation choices we made using the Shapley value solution concept have proved that parental favoritism does exist in most cases of the brood data. The restricted structure of the brood datasets also enables the Shapley value solution concept to give a reasonable fit within a reasonable time. The brood data we are taking, the results of the experiments have tendency that small increase on the food allocation for the marginal could increase the chick's probability of survive a lot more, while giving more food to the core chick who already has a high survival rate does not give a different output as the core already has a great chance of surviving. To summarize the results of the experiments, the better the game fits the solutions, the more we can trust the resulting system. Shapley value solution concept to tackle the chick brood food allocation problem. We also successfully translate the biological problem of chick's food allocation into a cooperative game approach using various techniques known in literature.

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