ORIGINAL ARTICLE

# EDUCATIONAL APPROACH FOR THE UNDERSTANDING OF MAGNETIC INDUCTION BEHAVIOR FROM A 3-D CYLINDRICAL SOLENOID

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## ABSTRACT

**Objective**: This work aims to analytically compute the magnetic induction from a cylindrical solenoid. The approach is used to validate the work of graduate students in electrical engineering laboratory, University of Antsiranana, Madagascar. **Methods**: Based on the Biot-Savart law, the theoretical expressions of the tridimensional (3D) magnetic field components close to a metallic solenoid were established. **Results**: The effects of variations about solenoid's geometrical parameters (excentricity for instance) on the magnetic induction are illustrated. The proposed approach is illustrated throughout computing with numerical tools.

**Keywords:** *electrical engineering, electromagnetic compatibility, analytical approach, numerical modeling magnetic induction, cylindrical solenoid, Biot-Savart law.*

## 1.INTRODUCTION

Electromagnetics stands for funding knowledge for electrical engineering [1,2]. The electrical machine principle is an important part of this field of engineering as underlined in [3,4]. We are teaching the electrical machine functioning principle in the Faculty of Science of the University of Antsiranana (Madagascar) by combining the electric circuit theory and electromagnetism [5,6]. One of the main lectures for graduate students with laboratory work is the applications with transformers [7] and synchronous machines [8]. Moreover, we are dealing also on the further understanding about the different phenomena occurred between the electrical machines and the power electronic devices as the signal harmonics [9]. But for the basic understanding to this electromagnetic concept, the fundamental analytical theory is necessary. Relying on the effectiveness of the Biot-Savart assumption, the magnetic vector induction may be predicted from electrical wires at different points in the 3D zone. The modulus and orientation of the field can be then extracted based on the analytical formulas in function of the electrical structure geometrical parameters. It can be wondered about the uniformity of the magnetic field B for example across the circular coil propagated by current with intensity I. The cylindrical solenoid is among the shape of the reel most used in electrical engineering and electronics. The study of the induced magnetic fields may also be of great interests for electromagnetic compatibility (EMC) purposes. In the present paper, we address a fast analytical approach to help the electrical engineering students to understand the apparition and the variation of the static magnetic induction due to the circular conductors. As application examples, the proposed analysis is focused to the typically circular solenoid.

## 1.2 Analytical Approach for the Determination of Magnetic Induction 3D-Components

### 1.2.1 Magnetic Induction from a Single Circular Spire Element :

Figure 1 represents the overview of the single circular spire under consideration. The circular coil is referenced with the cylindrical system (O, $\vec{u_r}$ , $\vec{k}$ ) its center O(0,0,0) and a revolution axis (Oz). The coil radius was assumed equal to R and through which is propagating the current i.

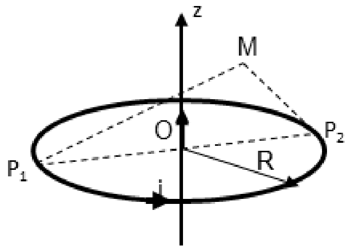

**Figure 1:** Representation of circular coil with radius *R*.

Based on the positive direction of this current, the magnetic induction $\vec{B}(M) = B_r \cdot \vec{u_r} + B_z \cdot \vec{k}$ at the arbitrary point *M*(*r*,*z*). At this observation point *M* which is defined by $\overrightarrow{OM} = r \cdot \vec{u_r} + z \cdot \vec{k}$, with our electrical engineering students, we propose to determine the magnetic induction based on the Biot-Savart law.

To do this, let us consider two points $P_1$ and $P_2$ diametrically opposed on the coil, such as $P_1$ is located compared to a fixed diameter by an angle *θ* Around $P_1$ consider an angular variation *dθ* to have the element of current $i \cdot R \cdot d\theta \cdot \vec{t}$ corresponding to $P_2$ $-i \cdot R \cdot d\theta \cdot \vec{t}$. The circular coil crossed by a current *i* created in a point two components, the radial component and the component along the axis of (*Oz*):

$$B_r(r,z) = \frac{\mu_0 \cdot i \cdot R \cdot z}{4}\left[\left((r-R)^2+z^2\right)^{-3/2} - \left((r+R)^2+z^2\right)^{-3/2}\right] \quad (1)$$

$$B_z(r,z) = \frac{\mu_0 \cdot i \cdot R}{4}\left[\frac{r+R}{\left((r+R)^2+z^2\right)^{3/2}} - \frac{r-R}{\left((r-R)^2+z^2\right)^{3/2}}\right]. \quad (2)$$

It implies that the total magnetic field modulus at any point M can be determined by $B(r,z) = \sqrt{B_r^2(r,z) + B_z^2(r,z)}$.

**1.2.2 Magnetic Induction from Solenoid :** Figures 2 and 3 depict the illustrative views of the solenoid under study. One considers a cylindrical solenoid of radius R and height 2l which is fed by a current of intensity i. We are proposing to determine the magnetic induction at the arbitrary point M(r,z). The number of coils per unit of length is n=N/(2l).

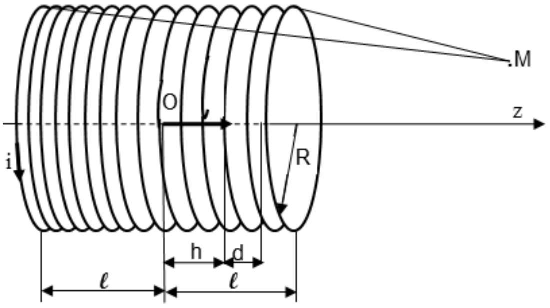

**Figure 3:** Perspective view of the cylindrical solenoid having *Oz* revolution axis.

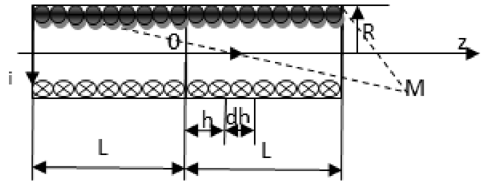

**Figure 4:** Top view of the cylindrical solenoid having *Oz* revolution axis.

Around the point located at the height *h* from the center O, we have the current $di = n \cdot i \cdot dh$ through the elementary coils thickness *dh*. Based on relations (1) and (2), the induced field from an elementary part of coil (width *dh*) can be expressed in point M as:

$$dB_r = \frac{\mu_0 \cdot n \cdot i \cdot R \cdot dh}{4}\left(\frac{Z-h}{\left((r-R)^2+(Z-h)^2\right)^{\frac{3}{2}}} - \frac{Z-h}{\left((r+R)^2+(Z-h)^2\right)^{\frac{3}{2}}}\right), \quad (3)$$

$$dB_z = \frac{\mu_0 \cdot R \cdot di}{4}\left(\frac{(r+R)}{\left((r+R)^2+(Z-h)^2\right)^{\frac{3}{2}}} - \frac{(r-R)}{\left((r-R)^2+(Z-h)^2\right)^{\frac{3}{2}}}\right). \quad (4)$$

The cylindrical solenoid traversed by a current of intensity *i* generates in point M two components, the radial component and the component along axis (*Oz*).

$$B_r(r,z) = \frac{\mu_0 \cdot n \cdot i \cdot R}{4}\left[\psi_1(r,z) - \psi_2(r,z)\right], \quad (5)$$

where

$$\psi_1(r,z) = \frac{1}{\sqrt{(r-R)^2+(z-l)^2}} - \frac{1}{\sqrt{(r-R)^2+(z+l)^2}}, \quad (6)$$

$$\psi_2(r,z)=\frac{1}{\sqrt{(\mathrm{r}+\mathrm{R})^2+(\mathrm{z}-\mathrm{l})^2}}-\frac{1}{\sqrt{(\mathrm{r}+\mathrm{R})^2+(\mathrm{z}+\mathrm{l})^2}}, \quad \textbf{(7)}$$

Similarly, the magnetic induction vertical component is expressed as:

$$B_z(r,z)=\frac{\mu_0\cdot n\cdot i\cdot R}{4}\left[\psi_3(r,z)-\psi_4(r,z)\right], \quad \textbf{(8)}$$

Wiith :

$$\psi_3(r,z)=\frac{1}{(r+R)}\left[\frac{z+l}{\sqrt{(r+R)^2+(z+l)^2}}-\frac{z-l}{\sqrt{(r+R)^2+(z-l)^2}}\right], \quad \textbf{(9)}$$

$$\psi_4(r,z)=\frac{1}{(r-R)}\left[\frac{z+l}{\sqrt{(r-R)^2+(z+l)^2}}-\frac{z-l}{\sqrt{(r-R)^2+(z-l)^2}}\right]. \quad \textbf{(10)}$$

## 2. RESULTS AND DISCUSSION

For the further insight on the analytical approach introduced previously, numerical computations were conducted by considering arbitrary solenoid parameters. The following paragraphs summarize the obtained results.

### 2.1 Analytical Investigation on the Magnetic Induction from a Single Coil

For the numerical testing, a single metallic coil with radius *R*=2mm is considered in this paragraph. The components of magnetic induction are extracted from formulas (2) and (3).

In the first case, the ratio of the components of magnetic induction ($B_r$(z), $B_z$(z) and B(z)) over current I are computed by varying z from -6mm to 6mm. The obtained results are displayed in Figure 4. It can be pointed out that the total induction B(z) is maximal when *z*=0. However, $B_z$ reaches its maximal values for z=0.72m.
Then, we also plotted the slice cuts defined by $x_{min}$=-4mm, $x_{max}$=4mm, $y_{min}$=-4mm and $y_{max}$=4mm of the total magnetic inductance in the horizontal planes placed above spires. Figure 5 displays the computed results at z={1.5mm, 2.0mm, 2.5mm, 3.0mm}. These cartographies and the surface plotting in Figure 6 explain the remoteness influence on the magnetic induction.

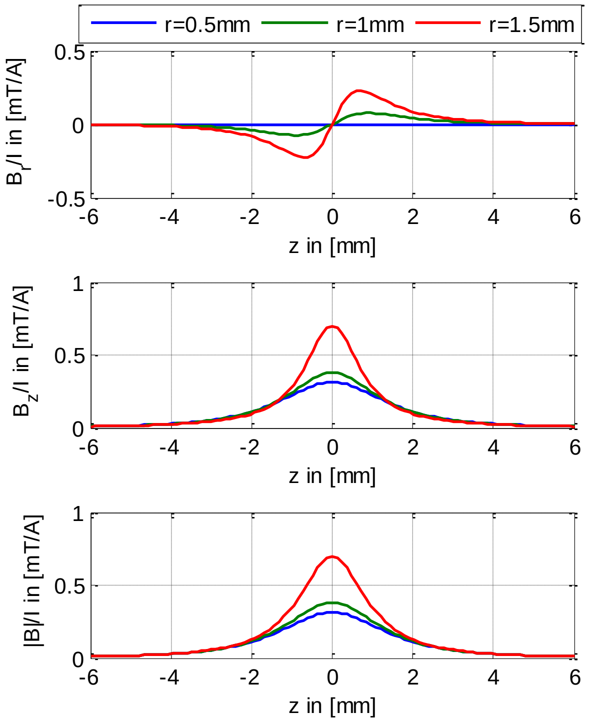

**Figure 6:** Variation of $B_z$ and $B_r$ along the axis defined by r=1mm.

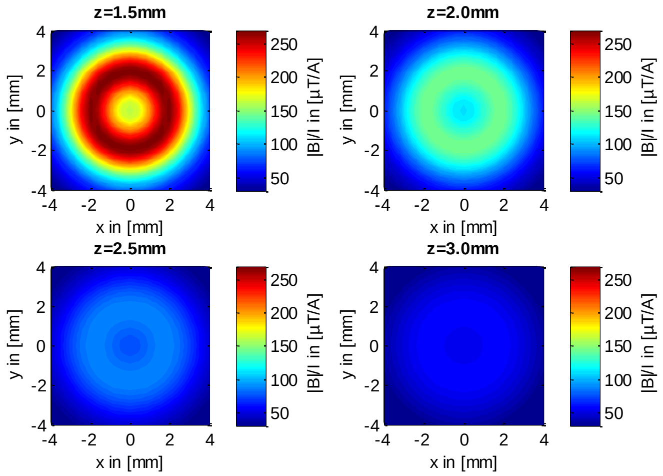

**Figure 7:** Computed magnetic induction maps in the planes z={1.5mm,2.0mm,2.5mm,3.0mm} above of the solenoid center.

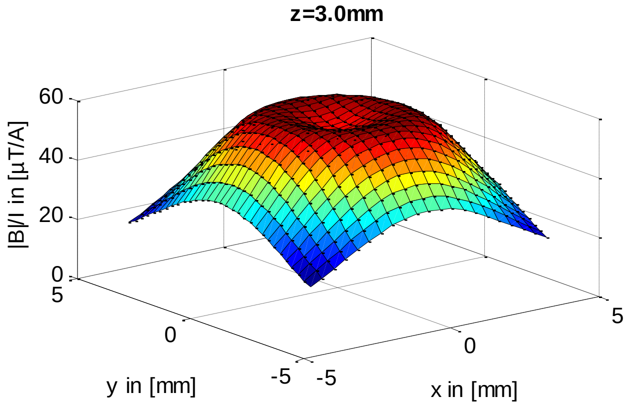

**Figure 8:** Surface plot of the magnetic induction at z=3mm above of the solenoid center illustrating the excentricity effect.

**2.3 Magnetic Induction Computed Along the Solenoid Radial Axis :** During the numerical computations, a circular solenoid presenting N=100 coils, radius R=2mm and length l=6mm was considered. Furthermore, through this structure was supposed propagating a direct current with intensity I=1A. By varying the distances z and r, we obtained the family of the radial magnetic induction $B_r(r,z)$ curves plotted in Figure 7, thanks to equation (5). As expected, it can be emphasized that the magnetic induction intensity presents a symmetrical aspect through the median plane of the solenoid. Moreover, induction peaks are localized at the solenoid extremities.

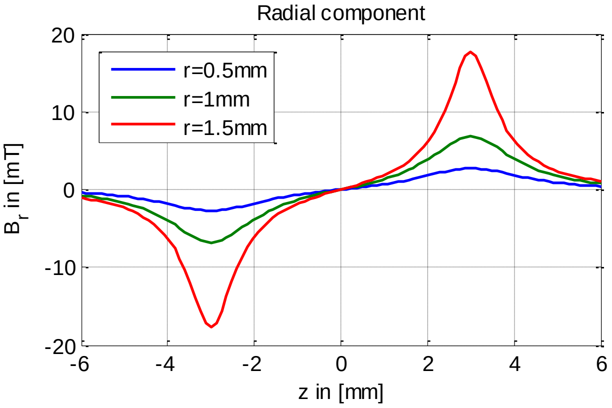

**Figure 9:** Computed magnetic induction along the radial axis of the circular solenoid versus z.

## 2.4 Computed Magnetic Induction Along the Solenoid Revolution Axis

With the same test structure, we also calculated analytically the magnetic induction vertical component based on equation (8). After numerical computation, we obtained the curves of $B_z(r,z)$ plotted in Figure 8. As expected the z-component induction is maximal when $z$=0. Then, its intensity dropped rapidly when the distance $r$ is higher.

In order to propose comparisons with realistic test cases, the final paper will be completed with 3D-simulations and experimental measurements.

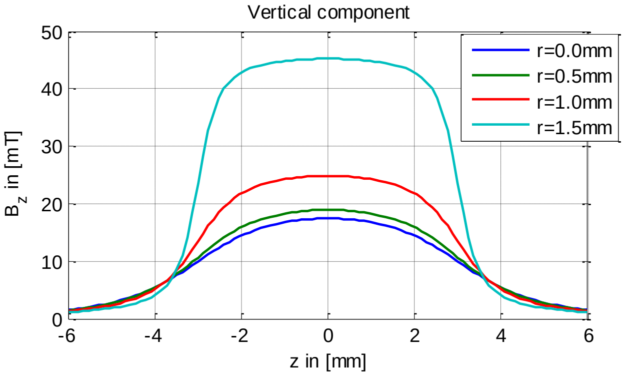

**Figure 10:** Computed magnetic induction vertical components of the circular solenoid versus z.

## 4. CONCLUSION AND FUTURE WORK

An analytical approach for calculating the magnetic induction components at any point M in proximity of the solenoid was presented. The radial component (usually considered to hold much part of the magnetic state modification) is considered with a serious care here.

- A classical assumption is often to consider the magnetic induction is uniform on the entire wolume of the reer. Figure 4 clearly justifies the previous assumption is restricted close to a coil. Extending the previous hypothesis to the whole solenoid will lead to huge errors (Figures 7 and 8).
- On the didactic level, the trainins students in electrotechnoly and electronics may be improved by demonstrating the importance of considering both radial and longitudinal magnetic components in their future daily work The proposed study makes last aims easier.

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